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New Physics and CP Violation in Hyperon Nonleptonic Decays

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Abstract

The sum of the CP-violating asymmetries $A(\Lambda^0_-)$ and $A(\Xi^-_-)$ in hyperon nonleptonic decays is presently being measured by the E871 experiment. We evaluate contributions to the asymmetries induced by chromomagnetic-penguin operators, whose coefficients can be enhanced in certain models of new physics. Incorporating recent information on the strong phases in $\Xi \to \Lambda\pi$ decay, we show that new-physics contributions to the two asymmetries can be comparable. We explore how the upcoming results of E871 may constrain the coefficients of the operators. We find that its preliminary measurement is already better than the ϵ parameter of K-$\bar{K}$ mixing in bounding the parity-conserving contributions.

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I. INTRODUCTION

The phenomenon of CP violation remains one of the least understood aspects of particle physics. Although CP violation has now been detected in several processes in the kaon and B-meson systems [1], its origin is still far from clear. The standard model (SM) can accommodate the experimental results, but they do not yet provide critical tests for it [2]. To pin down the sources of CP violation within or beyond the SM, it is essential to observe it in many other processes.

Nonleptonic hyperon decays provide an environment where it is possible to obtain additional observations of CP violation. Although this has been recognized for a long time [3], only recently has it been experimentally feasible to search for CP violation in some of these decays [4–6]. There is currently such an effort being done at Fermilab, by the HyperCP (E871) Collaboration [5, 6].

The reactions studied by HyperCP are the decay sequence $\Xi^- \to \Lambda\pi^-$, $\Lambda \to p\pi^-$ and its antiparticle counterpart. For each of these decays, the decay distribution in the rest frame of the parent hyperon with known polarization $\boldsymbol{w}$ has the form

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} \sim 1 + \alpha\, \boldsymbol{w}\cdot\hat{\boldsymbol{p}} \;, \tag{1}$$

where $\hat{\boldsymbol{p}}$ is the unit vector of the daughter-baryon momentum and α is the parameter relevant to the CP violation of interest. By evaluating the decay chain $\Xi \to \Lambda\pi \to p\pi\pi$, the HyperCP experiment is sensitive to the *sum* of CP violation in the Ξ decay and CP violation in the Λ decay. Thus it measures [5, 6]

$$A_{\Xi\Lambda} \;=\; A_\Lambda + A_\Xi \;, \tag{2}$$

where

$$A_\Xi \;\equiv\; A\big(\Xi^-_-\big) \;\equiv\; \frac{\alpha_\Xi + \overline{\alpha}_\Xi}{\alpha_\Xi - \overline{\alpha}_\Xi} \;, \qquad\qquad A_\Lambda \;\equiv\; A\big(\Lambda^0_-\big) \;\equiv\; \frac{\alpha_\Lambda + \overline{\alpha}_\Lambda}{\alpha_\Lambda - \overline{\alpha}_\Lambda} \;, \tag{3}$$

are the CP-violating asymmetries in $\Xi \to \Lambda\pi$ and $\Lambda \to p\pi$, respectively. The published measurements currently available [7] are $A_\Lambda = 0.012 \pm 0.021$ and $A_{\Xi\Lambda} = 0.012 \pm 0.014$. HyperCP will obtain more precise results, with an expected sensitivity of $\sim\!10^{-4}$, and has recently reported [6] a preliminary measurement of $A_{\Xi\Lambda} = (-7 \pm 12 \pm 6.2) \times 10^{-4}$.

The amplitudes for both $\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$ contain S- and P-wave components, each of which consists of contributions describing $|\Delta I| = \frac{1}{2}$ and $\frac{3}{2}$ transitions, with the former being known to dominate. Assuming $|\Delta I| = \frac{1}{2}$ dominance, one can derive at leading order [8]

$$A_\Lambda \;=\; -\tan\big(\delta_P^\Lambda - \delta_S^\Lambda\big)\sin\big(\phi_P^\Lambda - \phi_S^\Lambda\big) \;, \qquad\qquad A_\Xi \;=\; -\tan\big(\delta_P^\Xi - \delta_S^\Xi\big)\sin\big(\phi_P^\Xi - \phi_S^\Xi\big) \;. \tag{4}$$

Here, δ_S^Λ $\big(\delta_P^\Lambda\big)$ is the strong S-wave (P-wave) $N\pi$ scattering phase-shift at $\sqrt{s} = M_\Lambda$, and δ_S^Ξ $\big(\delta_P^\Xi\big)$ is the strong S-wave (P-wave) $\Lambda\pi$ scattering phase-shift at $\sqrt{s} = M_\Xi$. Moreover, $\phi_S^{\Lambda,\Xi}$ $\big(\phi_P^{\Lambda,\Xi}\big)$ are the CP-violating weak phases induced by the $|\Delta S| = 1$, $|\Delta I| = \frac{1}{2}$ interaction in the S-wave (P-wave) of the $\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$ decays, respectively.

The strong $N\pi$ scattering phases needed in Eq. (4) have been measured [9] to be $\delta_S^\Lambda \sim 6^\circ$ and $\delta_P^\Lambda \sim -1^\circ$ with errors of about 1°. In contrast, the strong $\Lambda\pi$ phases are less well determined. Using the current PDG numbers [7], one can deduce [10] the phase difference $\delta_P^\Xi - \delta_S^\Xi = (-7.7 \pm 7.7)^\circ$.

Very recently, the E756 Collaboration [11] has published a new measurement of $\delta_P^\Xi - \delta_S^\Xi = (+3.17 \pm 5.28 \pm 0.73)^\circ$. HyperCP is presently also measuring this quantity, with better precision, and has reported [10] a preliminary result of $\delta_P^\Xi - \delta_S^\Xi = \bigl(7.6 \pm 1.3^{+2.4}_{-2.8}\bigr)^\circ$.

On the theoretical side, the most recent update [12] of the standard-model prediction of $A_{\Xi\Lambda}$ yields a value that is smaller than most of earlier estimates [8, 13], but with a sizable uncertainty, resulting in the range $\left|A_{\Xi\Lambda}\right| \lesssim 5 \times 10^{-5}$, which is compatible with some of the earlier predictions. Thus, the upcoming data from HyperCP will likely be insensitive to SM effects. Beyond the SM, the asymmetry is potentially more detectable, as various estimates [14, 15] indicate that A_Λ could be as large as 10^{-3} in models with enhanced chromomagnetic-penguin operators (CMO). In these studies, the corresponding value of A_Ξ has been neglected because most of recent calculations based on chiral perturbation theory [16, 17] suggest that $\delta_P^\Xi - \delta_S^\Xi$ is small compared to $\delta_P^\Lambda - \delta_S^\Lambda$. However, there are early indications, from a coupled-channel K-matrix estimate given in Ref. [17] and from the preliminary result of HyperCP above [10], that the two phase-differences may actually be comparable in size. This is also compatible with the other two measurements of $\delta_P^\Xi - \delta_S^\Xi$ mentioned earlier. Since HyperCP is sensitive only to the sum $A_\Lambda + A_\Xi$, it is therefore important to have an up-to-date expectation of $A_{\Lambda,\Xi}$ from possible new physics and of their sum.

In this paper, we estimate both A_Λ and A_Ξ due to possible physics beyond the SM, incorporating the new information on the strong phases and taking into account constraints from kaon data. Specifically, we consider contributions generated by the CMO, which could be significantly larger that their SM counterparts [14, 15]. The relevant effective Hamiltonian can be written as [18]

$$\mathcal{H}_{\rm w} \,=\, C_g\, Q_g \,+\, \tilde{C}_g\, \tilde{Q}_g \,\,+\,\, {\rm H.c.}\,, \tag{5}$$

where C_g and $\tilde{C}_g$ are the Wilson coefficients, and

$$Q_g \,=\, \frac{g_{\rm s}}{16\pi^2}\, \bar{d}\, \sigma^{\mu\nu} t^a \left(1+\gamma_5\right) s\, G^a_{\mu\nu}\,, \qquad \tilde{Q}_g \,=\, \frac{g_{\rm s}}{16\pi^2}\, \bar{d}\, \sigma^{\mu\nu} t^a \left(1-\gamma_5\right) s\, G^a_{\mu\nu}\,, \tag{6}$$

are the CMO, with $G_a^{\mu\nu}$ being the gluon field-strength tensor and ${\rm Tr}\bigl(t^a t^b\bigr) = \frac{1}{2}\delta^{ab}$. These operators also contribute to the CP-violating parameters ϵ in kaon mixing and ϵ' in kaon decay, as well as to other hyperon and kaon observables [18–20]. Although ϵ, ϵ', and $A_{\Lambda,\Xi}$ receive contributions from the same $|\Delta S| = 1$ interaction, they probe different parts of it. Whereas ϵ and ϵ' are sensitive only to parity-even and parity-odd contributions, respectively, $A_{\Lambda,\Xi}$ are sensitive to both. Thus, with ϵ and ϵ' now being well measured, we will estimate the range of $A_{\Xi\Lambda}$ arising from the CMO that is allowed by ϵ and ϵ', and then compare it with the preliminary result of HyperCP mentioned above. Since various new-physics scenarios may contribute differently to the coefficients of the operators, we will not focus on specific models, but will instead adopt a model-independent approach, only assuming that the contributions are potentially sizable. Accordingly, we will also explore how well the coefficients can be constrained in the event that HyperCP detects no CP-violation.

In Sec. II, we employ heavy-baryon chiral perturbation theory to derive the decay amplitudes at leading order. In Sec. III, we calculate the weak phases using matrix elements estimated in the MIT bag model. We then estimate the CP-violating asymmetries, taking into account constraints from CP violation in the kaon system, and present a discussion of our results. We give our conclusions in Sec. IV.

II. CHIRAL LAGRANGIAN AND DECAY AMPLITUDES

The chiral Lagrangian that describes the interactions of the lowest-lying mesons and baryons is written down in terms of 3×3 matrices φ and B which contain the lightest meson-octet and baryon-octet fields, respectively [21, 22]. The mesons enter through the exponential $\Sigma=\xi^2=\exp(\mathrm{i}\varphi/f)$, where f is the pion-decay constant.

In the heavy-baryon formalism [22], the baryons in the chiral Lagrangian are described by velocity-dependent fields, B_v. For the strong interactions, the chiral Lagrangian to lowest order in the derivative expansion is given by [22]

$$\mathcal{L}_{\rm s}^{(1)} \,=\, \big\langle\bar{B}_v\,\mathrm{i}v\cdot\mathcal{D}B_v\big\rangle + 2D\,\big\langle\bar{B}_vS_v^\mu\left\{\mathcal{A}_\mu,B_v\right\}\big\rangle + 2F\,\big\langle\bar{B}_vS_v^\mu\left[\mathcal{A}_\mu,B_v\right]\big\rangle \,+\, \tfrac{1}{4}f^2\left\langle\partial^\mu\Sigma^\dagger\,\partial_\mu\Sigma\right\rangle \ , \tag{7}$$

where $\langle\cdots\rangle$ denotes $\mathrm{Tr}(\cdots)$ in flavor-SU(3) space, S_v is the spin operator, $\mathcal{D}^\mu B_v=\partial^\mu B_v+\left[\mathcal{V}^\mu,B_v\right]$, with $\mathcal{V}_\mu=\frac{1}{2}\left(\xi\,\partial_\mu\xi^\dagger+\xi^\dagger\,\partial_\mu\xi\right)$, and $\mathcal{A}_\mu=\frac{\mathrm{i}}{2}\left(\xi\,\partial_\mu\xi^\dagger-\xi^\dagger\,\partial_\mu\xi\right)$. In this Lagrangian, D and F are free parameters which can be determined from hyperon semileptonic decays. We will adopt the parameter values obtained from fitting tree-level formulas [22], namely $D=0.80$ and $F=0.50$. We will also need the chiral Lagrangian that explicitly break chiral symmetry [23], containing one power of the quark-mass matrix $M=\mathrm{diag}\big(m_u,m_d,m_s\big)$,

$$\mathcal{L}_{\rm s}^{(2)} \,=\, \tfrac{1}{4}f^2\left\langle\chi_+\right\rangle \,+\, \frac{b_D}{2B_0}\left\langle\bar{B}_v\left\{\chi_+,B_v\right\}\right\rangle + \frac{b_F}{2B_0}\left\langle\bar{B}_v\left[\chi_+,B_v\right]\right\rangle + \frac{b_0}{2B_0}\left\langle\chi_+\right\rangle\left\langle\bar{B}_vB_v\right\rangle \ , \tag{8}$$

where we have used the notation $\chi_+=\xi^\dagger\chi\xi^\dagger+\xi\chi^\dagger\xi$ to introduce coupling to external (pseudo)scalar sources, $\chi=s+ip$, such that in the absence of the external sources χ reduces to the mass matrix, $\chi=2B_0M$. We will take the isospin limit, $m_u=m_d=\hat{m}$, and consequently $\chi=\mathrm{diag}\big(m_\pi^2,m_\pi^2,2m_K^2-m_\pi^2\big)$. In Eq. (8), the constants B_0, $b_{D,F,0}$ are free parameters which can be fixed from data.

In the weak sector, the chiral Lagrangian induced by the chromomagnetic-penguin operators has to respect their symmetry properties. From Eq. (5), we observe that Q_g and $\tilde{Q}_g$ transform as $\big(\bar{3}_{\rm L},3_{\rm R}\big)$ and $\big(3_{\rm L},\bar{3}_{\rm R}\big)$, respectively, under $\mathrm{SU(3)_L\times SU(3)_R}$ transformations. Accordingly, the desired Lagrangian at leading order is

$$\begin{aligned}\mathcal{L}_{\rm w} \,=\,& \beta_D\left\langle\bar{B}_v\left\{\xi^\dagger h\xi^\dagger,B_v\right\}\right\rangle + \beta_F\left\langle\bar{B}_v\left[\xi^\dagger h\xi^\dagger,B_v\right]\right\rangle + \beta_0\left\langle h\Sigma^\dagger\right\rangle\left\langle\bar{B}_vB_v\right\rangle \,+\, \beta_\varphi\,f^2B_0\left\langle h\Sigma^\dagger\right\rangle \\ &+\,\tilde{\beta}_D\left\langle\bar{B}_v\left\{\xi h\xi,B_v\right\}\right\rangle + \tilde{\beta}_F\left\langle\bar{B}_v\left[\xi h\xi,B_v\right]\right\rangle + \tilde{\beta}_0\left\langle h\Sigma\right\rangle\left\langle\bar{B}_vB_v\right\rangle \,+\, \tilde{\beta}_\varphi\,f^2B_0\left\langle h\Sigma\right\rangle \,+\, \mathrm{H.c.}\ ,\end{aligned} \tag{9}$$

where β_i $\big(\tilde{\beta}_i\big)$ are parameters containing the coefficient C_g $\big(\tilde{C}_g\big)$, and the 3×3-matrix h selects out $s\to d$ transitions, having elements $h_{kl}=\delta_{k2}\delta_{3l}$. As shown in Appendix A, the expression for $\mathcal{L}_{\rm w}$ can be inferred from the lowest-order chiral realization of the densities $\bar{d}(1\pm\gamma_5)s$. We remark that this Lagrangian is of $\mathcal{O}(1)$ in the derivative and m_s expansions.

For the weak decay of a spin-$\frac{1}{2}$ baryon B into another spin-$\frac{1}{2}$ baryon B' and a pseudoscalar meson ϕ, the amplitude in the heavy-baryon approach has the general form [24]

$$\mathrm{i}\mathcal{M}_{B\to B'\phi} \,=\, -\mathrm{i}\big\langle B'\phi\big|\mathcal{L}_{\rm w+s}\big|B\big\rangle \,=\, \bar{u}_{B'}\left(\mathcal{A}_{BB'\phi}^{(S)}+2S_v\cdot p_\phi\,\mathcal{A}_{BB'\phi}^{(P)}\right)u_B\ , \tag{10}$$

where the superscripts refer to the S- and P-wave components of the amplitude. These components are related to the decay width Γ and parameter α by

$$\Gamma = \frac{|\boldsymbol{p}_{B'}|}{4\pi\, m_B}\big(E_{B'} + m_{B'}\big)\left(|s|^2 + |p|^2\right) , \qquad \alpha = \frac{2\,\mathrm{Re}(s^* p)}{|s|^2 + |p|^2} , \tag{11}$$

where $s = \mathcal{A}^{(S)}$ and $p = |\boldsymbol{p}_{B'}|\,\mathcal{A}^{(P)}$. To express our results, we also adopt the notation [24]

$$a_{BB'\phi}^{(S,P)} \equiv \sqrt{2}\, f\, \mathcal{A}_{BB'\phi}^{(S,P)} . \tag{12}$$

From the Lagrangians given above, one can derive the S- and P-wave amplitudes at leading order, represented by the diagrams in Figs. 1 and 2, respectively. For the S-wave, the first diagram is directly obtained from a weak vertex provided by Eq. (9), whereas the other diagram involves a $\bar{K}$-vacuum tadpole from Eq. (9) and a strong $B \to B'\bar{K}\pi$ vertex, which consists of contributions from both $\mathcal{L}_\mathrm{s}^{(1)}$ and $\mathcal{L}_\mathrm{s}^{(2)}$. It is worth mentioning here that the β_φ and $\tilde{\beta}_\varphi$ terms in $\mathcal{L}_\mathrm{w}$ do not contribute to $\bar{K} \to \pi\pi$ decay, as the corresponding direct and tadpole diagrams cancel exactly [25]. For $\Lambda \to p\pi^-$ and $\Xi^- \to \Lambda\pi^-$, the resulting amplitudes are then[1]

$$\begin{aligned} a_{\Lambda p\pi^-}^{(S)} &= \tfrac{1}{\sqrt{6}}\left(\beta_D^- + 3\beta_F^-\right) + \tfrac{3}{\sqrt{6}}\,\beta_\varphi^-\,\frac{m_\Lambda - m_N}{m_s - \hat{m}} , \\ a_{\Xi^-\Lambda\pi^-}^{(S)} &= \tfrac{1}{\sqrt{6}}\left(\beta_D^- - 3\beta_F^-\right) - \tfrac{3}{\sqrt{6}}\,\beta_\varphi^-\,\frac{m_\Xi - m_\Lambda}{m_s - \hat{m}} , \end{aligned} \tag{13}$$

where $\beta_i^- \equiv \beta_i - \tilde{\beta}_i$ and we have used the relations

$$\begin{gathered} m_\Lambda - m_N = -\tfrac{2}{3}\big(b_D + 3b_F\big)(m_s - \hat{m}) , \qquad m_\Xi - m_\Lambda = \tfrac{2}{3}\big(b_D - 3b_F\big)(m_s - \hat{m}) , \\ m_K^2 = B_0\,(m_s + \hat{m}) , \end{gathered} \tag{14}$$

derived from Eq. (8). For the P-wave, the amplitude arises from two baryon-pole diagrams, each involving a weak vertex from Eq. (9) and a strong vertex from Eq. (7), and a kaon-pole diagram involving a strong vertex from Eq. (7) followed by a $\bar{K}$-π vertex from Eq. (9). Thus we find

$$\begin{aligned} a_{\Lambda p\pi^-}^{(P)} &= \frac{(D+F)\left(\beta_D^+ + 3\beta_F^+\right)}{\sqrt{6}\,\left(m_\Lambda - m_N\right)} + \frac{2D\left(\beta_D^+ - \beta_F^+\right)}{\sqrt{6}\,\left(m_\Sigma - m_N\right)} + \frac{(D+3F)\,\beta_\varphi^+}{\sqrt{6}\,(m_s - \hat{m})} , \\ a_{\Xi^-\Lambda\pi^-}^{(P)} &= \frac{(-D+F)\left(\beta_D^+ - 3\beta_F^+\right)}{\sqrt{6}\,\left(m_\Xi - m_\Lambda\right)} - \frac{2D\left(\beta_D^+ + \beta_F^+\right)}{\sqrt{6}\,\left(m_\Xi - m_\Sigma\right)} + \frac{(D-3F)\,\beta_\varphi^+}{\sqrt{6}\,(m_s - \hat{m})} , \end{aligned} \tag{15}$$

where $\beta_i^+ \equiv \beta_i + \tilde{\beta}_i$ and we have used $m_K^2 - m_\pi^2 = B_0\,(m_s - \hat{m})$. We note that the baryon and meson masses in all the amplitudes above are isospin-averaged ones.

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[1] The contribution from $\mathcal{L}_\mathrm{s}^{(1)}$ to the tadpole amplitude contains the factor $v\cdot p_\phi = m_B - m_{B'} = \mathcal{O}(m_s)$. As a result, the $\mathcal{L}_\mathrm{s}^{(1)}$ and $\mathcal{L}_\mathrm{s}^{(2)}$ contributions to the amplitude both have the same m_s order, $\mathcal{O}(m_s^0)$, as the β_φ^- terms indicate.

FIG. 1: Leading-order diagrams for chromomagnetic-penguin contributions to S-wave hyperon nonleptonic decays. In all figures, a solid (dashed) line denotes a baryon (meson) field, and a solid dot (hollow square) represents a strong (weak) vertex.

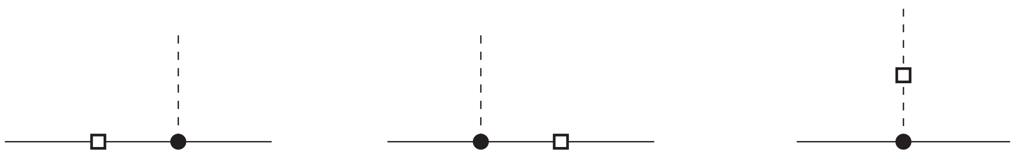

FIG. 2: Leading-order diagrams for chromomagnetic-penguin contributions to P-wave hyperon nonleptonic decays.

III. NUMERICAL RESULTS AND DISCUSSION

In order to estimate the weak phases in $A_{\Lambda,\Xi}$, we first need to determine the parameters β_i and $\tilde{\beta}_i$ in terms of the underlying coefficients C_g and $\tilde{C}_g$, respectively. From the effective Hamiltonian in Eq. (5) and the chiral Lagrangian in Eq. (9), we can derive the one-particle matrix elements

$$\begin{aligned} \langle n|\mathcal{H}_{\rm w}|\Lambda\rangle &= \frac{\beta_D+3\beta_F+\tilde{\beta}_D+3\tilde{\beta}_F}{\sqrt{6}}\,\bar{u}_n u_\Lambda \;, \\ \langle \Lambda|\mathcal{H}_{\rm w}|\Xi^0\rangle &= \frac{\beta_D-3\beta_F+\tilde{\beta}_D-3\tilde{\beta}_F}{\sqrt{6}}\,\bar{u}_\Lambda u_\Xi \;, \\ \langle \pi^-|\mathcal{H}_{\rm w}|K^-\rangle &= \big(\beta_\varphi+\tilde{\beta}_\varphi\big)\,B_0 \;. \end{aligned} \tag{16}$$

Since there is presently no reliable way to calculate these matrix elements from first principles, we will employ the MIT bag model to estimate them, following earlier work [15]. Using the results given in Appendix B, and setting $\tilde{C}_g=0$, we find

$$\beta_D \;=\; -\tfrac{3}{7}\,\beta_F \;=\; \frac{2\,I_M N^4}{\pi\,R^2}\,C_g \;, \qquad\qquad \beta_\varphi \;=\; \frac{-8\sqrt{2}\,I_M N^4\,m_K}{\pi B_0\,R^2}\,C_g \;, \tag{17}$$

where N, R, and I_M are bag parameters whose values are given in the appendix. By setting $C_g=0$ instead, one finds similar relations between $\tilde{\beta}_i$ and $\tilde{C}_g$. It follows that numerically

$$\overset{(\sim)}{\beta}_D = -\tfrac{3}{7}\,\overset{(\sim)}{\beta}_F \;=\; 1.10\times10^{-3}\;\overset{(\sim)}{C}_g\;\;{\rm GeV}^2 \;, \qquad\qquad \overset{(\sim)}{\beta}_\varphi = -3.49\times10^{-3}\;\overset{(\sim)}{C}_g\;\;{\rm GeV}^2 \;, \tag{18}$$

where we have used $B_0=m_K^2/(m_s+\hat{m})$, with $m_K=495.7\,{\rm MeV}$ and [2] $m_s+\hat{m}=121\,{\rm MeV}$. We note that C_g and $\tilde{C}_g$ here are the Wilson coefficients at the low scale $\mu=\mathcal{O}(1\,{\rm GeV})$ and

hence already contain the QCD running from the new-physics scales. We also note that the bag-model numbers in Eq. (18) are comparable in magnitude to the natural values of the parameters as obtained from naive dimensional analysis [26], e.g.,

$$\beta_{D,F}^{\rm NDA} = \frac{C_g\, g_{\rm s}}{16\pi^2}\,\frac{\Lambda^2}{4\pi} \sim 0.0024\, C_g\ {\rm GeV}^2 \ , \qquad \beta_{\varphi}^{\rm NDA} = \frac{C_g\, g_{\rm s}}{16\pi^2}\,\frac{\Lambda^3}{4\pi B_0} \sim 0.0014\, C_g\ {\rm GeV}^2 \ , \tag{19}$$

where $\Lambda = 4\pi f$ is the chiral-symmetry breaking scale, with $f = f_\pi \simeq 92.4\,{\rm MeV}$, and we have chosen $g_{\rm s} = \sqrt{4\pi}$. The differences between the two sets of numbers provide an indication of the level of uncertainty in estimating the matrix elements. This will be taken into account in our results below.

Next, we adopt the usual prescription for obtaining a weak phase [8, 12, 15], namely dividing the imaginary part of the theoretical amplitude by the real part of the amplitude extracted from experiment under the assumption of no CP violation. For the real part of amplitudes, we employ the experimental values obtained in Ref. [27]. They are, in units of $G_{\rm F} m_{\pi^+}^2$,

$$\begin{aligned} s_{\Lambda\to p\pi^-} &= 1.42 \pm 0.01 \ , & s_{\Xi^-\to\Lambda\pi^-} &= -1.98 \pm 0.01 \ , \\ p_{\Lambda\to p\pi^-} &= 0.52 \pm 0.01 \ , & p_{\Xi^-\to\Lambda\pi^-} &= 0.48 \pm 0.02 \ . \end{aligned} \tag{20}$$

The imaginary part of the amplitudes are calculated from Eqs. (13) and (15), combined with Eq. (18). The other hadron masses that we employ are $m_N = 938.9$, $m_\Lambda = 1115.7$, $m_\Sigma = 1193.2$, $m_\Xi = 1318.1$, and $m_\pi = 137.3$, all in units of MeV. In Table I, we have collected the results, divided by the central values of Eq. (20), in terms of C_g and $\tilde{C}_g$. We find that in each of the amplitudes the $\beta_\varphi^\pm$ terms are numerically larger than the $\beta_{D,F}^\pm$ terms, by up to a factor of four, and both contribute with the same sign.

TABLE I: Ratios of theoretical amplitude arising from chromomagnetic operators to experimental amplitude, for S- and P-wave transitions.

Decay mode	$\dfrac{{\rm Im}\, s}{s_{\rm expt}}$ (GeV)	$\dfrac{{\rm Im}\, p}{p_{\rm expt}}$ (GeV)
$\Lambda \to p\pi^-$	$-2.2\times10^5\,{\rm Im}\big(C_g-\tilde{C}_g\big)$	$-2.6\times10^5\,{\rm Im}\big(C_g+\tilde{C}_g\big)$
$\Xi^- \to \Lambda\pi^-$	$-1.9\times10^5\,{\rm Im}\big(C_g-\tilde{C}_g\big)$	$+1.1\times10^5\,{\rm Im}\big(C_g+\tilde{C}_g\big)$

Since the ratios in Table I follow from the leading-order amplitudes in χPT, the uncertainty of our prediction will come partly from our lack of knowledge about the higher-order terms which are presently incalculable. Various studies of hyperon processes in the context of χPT show that the leading nonanalytic contributions to amplitudes can be comparable to the lowest-order terms [21, 22, 24, 27, 28]. We expect that a similar situation occurs here, and consequently we also expect the uncertainty due to the higher-order contributions to be comparable to our leading-order estimate. To reflect this, as well as the uncertainty in estimating the matrix elements above, we assign an

error of 200% to each of these ratios. In Table II, we have listed the ratios as the weak phases,[2] along with their uncertainties, in terms of $C_g^{\pm} \equiv C_g \pm \tilde{C}_g$. Accordingly, C_g^+ and C_g^- correspond to parity-even and parity-odd transitions, respectively.

TABLE II: Weak phases generated by chromomagnetic operators and strong-phase differences, $\delta_S - \delta_P$.

Decay mode	$10^{-5}\,\phi_S$ (GeV)	$10^{-5}\,\phi_P$ (GeV)	$\delta_S - \delta_P$
$\Lambda \to p\pi^-$	$(-2.2 \pm 4.4)\,\mathrm{Im}\,C_g^-$	$(-2.6 \pm 5.2)\,\mathrm{Im}\,C_g^+$	$7^\circ \pm 2^\circ$
$\Xi^- \to \Lambda\pi^-$	$(-1.9 \pm 3.8)\,\mathrm{Im}\,C_g^-$	$(1.1 \pm 2.2)\,\mathrm{Im}\,C_g^+$	$-2^\circ \pm 6^\circ$

In Table II, we have also included the strong-phase differences. The number for $\Lambda \to p\pi^-$ results from the measured phases quoted in Sec. I. For $\Xi^- \to \Lambda\pi^-$, while awaiting a definitive measurement by HyperCP, we have adopted the range $-7.8^\circ < \delta_S^\Xi - \delta_P^\Xi < +3.9^\circ$ estimated in Ref. [17]. This range is compatible with the experimental values known to date, including the preliminary measurement by HyperCP mentioned in Sec. I.

From the results in Table II, it follows that the contributions of the CMO are

$$\begin{aligned} 10^{-4}\left(A_\Lambda\right)_g &= (3.5 \pm 7.0)\,\mathrm{Im}\,C_g^- + (-4.2 \pm 8.3)\,\mathrm{Im}\,C_g^+ \,, \\ 10^{-4}\left(A_\Xi\right)_g &= (-2.0 \pm 6.0)\,\mathrm{Im}\,C_g^- + (-1.2 \pm 3.4)\,\mathrm{Im}\,C_g^+ \,. \end{aligned} \tag{21}$$

where the numbers on the right-hand sides are all in units of GeV. This indicates that $(A_\Xi)_g$ is not negligible compared to $(A_\Lambda)_g$ and hence should be included in evaluating $A_{\Xi\Lambda}$. Summing the two asymmetries then yields

$$10^{-4}\left(A_{\Xi\Lambda}\right)_g = (2 \pm 13)\,\mathrm{Im}\,C_g^- + (-5 \pm 12)\,\mathrm{Im}\,C_g^+ \,, \tag{22}$$

the right-hand side being again in GeV. The errors we quote here are obviously not Gaussian, and simply indicate the ranges resulting from our calculation of the phases.

Since the CMO also contribute to the parameters ϵ' and ϵ in the kaon sector, it is possible to obtain a bound on their contribution to $A_{\Xi\Lambda}$ using the measured values of ϵ' and ϵ. The contribution to ϵ' can be written as [15, 18]

$$\left(\frac{\epsilon'}{\epsilon}\right)_g = \left(5.2 \times 10^5\,\mathrm{GeV}\right) B_G\,\mathrm{Im}C_g^- \,, \tag{23}$$

[2] We remark here that the central values of the numerical factors in front of $\mathrm{Im}\,C_g^{\pm}$ for $\phi_{S,P}^\Lambda$ in Table II are larger, by roughly a factor of two, than the corresponding numbers obtained in Ref. [15], which considers contributions from a generic supersymmetric model. The disagreement may be due mainly to a factor-of-two difference between the matrix elements in Eqs. (B1) and (B2) and those employed in Ref. [15].

where B_G parameterizes the hadronic uncertainty, and $m_s + \hat{m} = 121\,\mathrm{MeV}$ has been used [2]. The contribution to ϵ occurs through long-distance effects, and the simplest ones arise from π^0, η, and η' poles [25], yielding

$$(\epsilon)_g \;=\; -\Big(2.3\times 10^5\,\mathrm{GeV}\Big)\,\kappa\;\mathrm{Im}C_g^+ \;, \tag{24}$$

where κ quantifies the contributions of the different poles. Hence Eq. (22) can be rewritten as

$$\big(A_{\Xi\Lambda}\big)_g \;=\; \frac{0.04\pm 0.25}{B_G}\left(\frac{\epsilon'}{\epsilon}\right)_g \;+\; \frac{0.22\pm 52}{\kappa}\,(\epsilon)_g \;. \tag{25}$$

To estimate the range of $\big(A_{\Xi\Lambda}\big)_g$ allowed by the experimental values $|\epsilon| = (22.80\pm 0.13)\times 10^{-4}$ and $\mathrm{Re}(\epsilon'/\epsilon) = (16.6\pm 1.6)\times 10^{-4}$ [2, 7], we require

$$\left(\frac{\epsilon'}{\epsilon}\right)_g \;<\; 19\times 10^{-4} \;, \qquad\qquad |\epsilon|_g \;<\; 23\times 10^{-4} \;. \tag{26}$$

Consequently, adopting $0.5 < B_G < 2$ and $0.2 < |\kappa| < 1$, after Ref. [15], we find the bound

$$\big|A_{\Xi\Lambda}\big|_g \;<\; 97\times 10^{-4} \;. \tag{27}$$

The upper limit of this range is allowed by the published data [7], but is disfavored by the preliminary result of HyperCP [6] quoted in Sec. I, exceeding it by several sigmas. Since the number in Eq. (27) is dominated by the $(\epsilon)_g$ bound, we can then conclude that the available preliminary measurement by HyperCP already probes the parity-even contributions better than ϵ does.

Now, it is possible that HyperCP will in the end observe no CP-violation in $A_{\Xi\Lambda}$. In that event, the data can be used to estimate the bounds on both $\mathrm{Im}C_g^\pm$. To explore this possibility, we assume that HyperCP will be able to reach the expected sensitivity of 2×10^{-4} [6], and so we take this number as the upper limit for $A_{\Xi\Lambda}$. Moreover, since our result in Eq. (22) has large uncertainties, for illustrative purposes we use its central value in what follows. Barring significant cancellations between the $\mathrm{Im}C_g^\pm$ terms, we can consider three possible cases, (i) $\mathrm{Im}C_g^+ = 0$ and $\mathrm{Im}C_g^- \neq 0$, (ii) $\mathrm{Im}C_g^+ \neq 0$ and $\mathrm{Im}C_g^- = 0$, and (iii) $\mathrm{Im}C_g^+ \sim -\mathrm{Im}C_g^- \neq 0$. Consequently, requiring $|A_{\Xi\Lambda}|_g < 2\times 10^{-4}$, we obtain for these cases

$$\begin{gathered}
\text{(i)}\;\; \big|\mathrm{Im}C_g^-\big| \;\lesssim\; 1\times 10^{-8}\,\mathrm{GeV}^{-1} \;, \qquad\qquad \text{(ii)}\;\; \big|\mathrm{Im}C_g^+\big| \;\lesssim\; 4\times 10^{-9}\,\mathrm{GeV}^{-1} \;, \\
\text{(iii)}\;\; \big|\mathrm{Im}C_g^+\big| \;\sim\; \big|\mathrm{Im}C_g^-\big| \;\lesssim\; 3\times 10^{-9}\,\mathrm{GeV}^{-1} \;.
\end{gathered} \tag{28}$$

For comparison, the requirements in Eq. (26) from ϵ' and ϵ measurements imply

$$\big|\mathrm{Im}C_g^-\big| \;<\; 7.4\times 10^{-9}\,\mathrm{GeV}^{-1} \;, \qquad\qquad \big|\mathrm{Im}C_g^+\big| \;<\; 5.0\times 10^{-8}\,\mathrm{GeV}^{-1} \;. \tag{29}$$

Similar or stronger bounds on $\mathrm{Im}C_g^+$ may also be obtainable from $K\to 3\pi$, $\pi\ell^+\ell^-$, $\pi\gamma\gamma$ decays [20]. Therefore, even if it turns out that HyperCP eventually does not discover CP violation in hyperon decays, its data can be expected to provide stringent constraints on the coefficients of the CMO in new-physics models, at a level that is comparable to or better than the bounds coming from the kaon sector.

IV. CONCLUSION

We have evaluated the CP-violating asymmetries A_Λ and A_Ξ induced by the chromomagnetic-penguin operators, whose coefficients can be enhanced in some scenarios of new physics. Including recent information on the strong phases in $\Xi \to \Lambda\pi$ and adopting a model-independent approach, we have shown that $\left(A_\Xi\right)_g$, which was neglected in earlier studies, can be comparable to $\left(A_\Lambda\right)_g$. We have found that the upper limit of the sum of these asymmetries, $\left(A_{\Xi\Lambda}\right)_g$, as allowed by ϵ and ϵ' data is already disfavored by the preliminary measurement of $A_{\Xi\Lambda}$ by HyperCP. It follows that this preliminary data already imposes a constraint on the parity-even contributions of the operators that is stronger than the bound obtained from ϵ in kaon mixing. We have explored how well the upcoming results from HyperCP may bound the coefficients of the operators in the event of null results. In that case, the data will likely yield significant constraints that are comparable to or better than those provided by kaon measurements.

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APPENDIX A: CHIRAL REALIZATION OF $(\bar{3}_{\rm L}, 3_{\rm R})$ AND $(3_{\rm L}, \bar{3}_{\rm R})$ OPERATORS

The form of the weak Lagrangian in Eq. (9) can be inferred from the lowest-order chiral realization of the operators $\bar{d}(1+\gamma_5)s$ and $\bar{d}(1-\gamma_5)s$. The reason is that these densities, like the operators Q_g and $\tilde{Q}_g$ in Eq. (5), transform as $\left(\bar{3}_{\rm L}, 3_{\rm R}\right)$ and $\left(3_{\rm L}, \bar{3}_{\rm R}\right)$, respectively, under ${\rm SU(3)_L}\times{\rm SU(3)_R}$ rotations. Using $\mathcal{L}_s^{(2)}$ in Eq. (8), one can derive the correspondences [12]

$$\begin{aligned} -\bar{d}_{\rm L} s_{\rm R} \iff\; & b_D \left(\xi^\dagger B_v \bar{B}_v \xi^\dagger + \xi^\dagger \bar{B}_v B_v \xi^\dagger\right)_{32} + b_F \left(\xi^\dagger B_v \bar{B}_v \xi^\dagger - \xi^\dagger \bar{B}_v B_v \xi^\dagger\right)_{32} + b_0\, \Sigma_{32}^\dagger \left\langle \bar{B}_v B_v \right\rangle \\ & + \tfrac{1}{2} f^2 B_0\, \Sigma_{32}^\dagger \;, \end{aligned} \tag{A1}$$

$$\begin{aligned} -\bar{d}_{\rm R} s_{\rm L} \iff\; & b_D \left(\xi B_v \bar{B}_v \xi + \xi \bar{B}_v B_v \xi\right)_{32} + b_F \left(\xi B_v \bar{B}_v \xi - \xi \bar{B}_v B_v \xi\right)_{32} + b_0\, \Sigma_{32} \left\langle \bar{B}_v B_v \right\rangle \\ & + \tfrac{1}{2} f^2 B_0\, \Sigma_{32} \;, \end{aligned} \tag{A2}$$

where $q_{\rm L} = \frac{1}{2}(1-\gamma_5)q$ and $q_{\rm R} = \frac{1}{2}(1+\gamma_5)q$. The form in Eq. (9) then follows.

It is worth noting here that each of the S- and P-wave amplitudes in Eqs. (13) and (15) vanishes if we set

$$\beta_{D,F} = c\, b_{D,F} \;, \qquad \beta_\varphi = \frac{c}{2} \;, \qquad \tilde{\beta}_{D,F} = \tilde{c}\, b_{D,F} \;, \qquad \tilde{\beta}_\varphi = \frac{\tilde{c}}{2} \;, \tag{A3}$$

with c and $\tilde{c}$ being constants, and also use the relations in Eq. (14) as well as

$$m_\Sigma - m_N = 2\big(b_D - b_F\big)(m_s - \hat{m}) \;, \qquad m_\Xi - m_\Sigma = -2\big(b_D + b_F\big)(m_s - \hat{m}) \;, \tag{A4}$$

derived from Eq. (8). This satisfies the requirement implied by the Feinberg-Kabir-Weinberg theorem [29] that the operators $\bar{d}(1 \pm \gamma_5)s$ cannot contribute to physical amplitudes [30], and thus serves as a check for the formulas in Eqs. (13) and (15).

APPENDIX B: BAG-MODEL PARAMETERS

Here we provide the estimate in the MIT bag model of the matrix elements of the chromomagnetic operators contained in Eq. (16). The relevant calculations can be found in Refs. [31, 32]. We have

$$\begin{aligned} \big\langle n \big| g_{\rm s}\, \bar{d} \sigma^{\mu\nu} \lambda^a \big(1 \pm \gamma_5\big) s\, G^a_{\mu\nu} \big| \Lambda \big\rangle &= \frac{-16\sqrt{6}\; g_{\rm s}^2\, N^4 I_M}{R^2}\, \bar{u}_n u_\Lambda \,, \\ \big\langle \Lambda \big| g_{\rm s}\, \bar{d} \sigma^{\mu\nu} \lambda^a \big(1 \pm \gamma_5\big) s\, G^a_{\mu\nu} \big| \Xi^0 \big\rangle &= \frac{64\sqrt{6}\; g_{\rm s}^2\, N^4 I_M}{3\, R^2}\, \bar{u}_\Lambda u_\Xi \,, \end{aligned} \tag{B1}$$

$$\big\langle \pi^- \big| g_{\rm s}\, \bar{d} \sigma^{\mu\nu} \lambda^a \big(1 \pm \gamma_5\big) s\, G^a_{\mu\nu} \big| K^- \big\rangle = \frac{-64\, g_{\rm s}^2\, N^4 I_M}{R^2} \sqrt{2 m_K^2} \,, \tag{B2}$$

where $\lambda^a = 2t^a$, only the parity-conserving part of the $s \to dg$ operators contributes, and R, N, and I_M are bag-model parameters [32]. Numerically, we choose $g_{\rm s}^2 = 4\pi$, corresponding to $\alpha_{\rm s} = 1$, and adopt $R = 5.0\,{\rm GeV}^{-1}$ for the baryons and $R = 3.3\,{\rm GeV}^{-1}$ for the mesons [32]. Since the weak parameters β_i and $\tilde{\beta}_i$ belong to a Lagrangian which respects SU(3) symmetry [$\mathcal{L}_{\rm w}$ in Eq. (9)], in writing Eqs. (B1) and (B2) we have employed SU(3)-symmetric kinematics. Accordingly, we take $m_u = m_d = m_s = 0$ and use the formulas given in Ref. [32] to obtain $N = 2.27$ and $I_M = 1.63 \times 10^{-3}$ for both the baryons and mesons. Finally, we note that Eq. (B2), together with the relation $\langle \pi^- | Q_g | K^- \rangle = -\sqrt{2}\, \langle \pi^0 | Q_g | \bar{K}^0 \rangle$, leads to the matrix element [25, 33] $A_{\bar{K}\pi} \equiv \big\langle \pi^0 \big| \bar{d} \sigma^{\mu\nu} \lambda^a (1 + \gamma_5) s\, G^a_{\mu\nu} \big| \bar{K}^0 \big\rangle = +64\, g_{\rm s}\, N^4 I_M\, m_K / R^2 \simeq 0.4\,{\rm GeV}^3$.

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